Review Problems 2

Consider the following two matrices:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}; B = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 3 \end{bmatrix}$$

1) Describe the columnspace of each matrix, that is, the span of the columns of the matrix.

2) Consider the associated linear transformations T_A and T_B. Describe the domain, codomain, and range of these functions.

3) Describe T_A and T_B as functions: that is, if the input is, say, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ then what is the output?

4) Let $T_1 = T_A \circ T_B$. Find $[T_1]$, the matrix associated to T_1. (Hint: use matrix multiplication. Use the domain and codomain of these functions to determine in which order to multiply)

5) Describe T_1 as a function, as well as its domain, codomain, and range.

6) Let $T_2 = T_B \circ T_A$. Find $[T_2]$, the matrix associated to T_2.

7) Describe T_2 as a function, as well as its domain, codomain, and range.

8) Are any of T_A, T_B, T_1, T_2 one-to-one and/or onto?

9) Which of T_A, T_B, T_1, T_2 are invertible?

10) For the T_i that are invertible, give its domain, codomain, and range.

11) For the T_i that are invertible, find $[T_i]$.

12) For the T_i that are invertible, verify that $[T_i] \cdot [T_i]^{-1} = I_n$ and that $[T_i]^{-1} \cdot [T_i] = I_n$.

13) Find $A^{-1} \cdot T_1 \cdot B \cdot T_2 \cdot A^{-1}$, if it makes sense to do so.